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EXPLOITING COLLATERAL INFORMATION IN THE ESTIMATION OF ITEM PARAMETERS

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Abstract

When using item response theory (IRT) models in educational and psychological measurement, it is standard practice to estimate the operating characteristics of test items from examinees' item responses alone. This is the final report of a project that employed Bayesian and empirical Bayesian methods to exploit additional information that is often available about test items (e.g., format, content, or cognitive processing requirements) or about examinees (e.g., educational background or demographic status). Practical and theoretical results obtained in a series of research reports are summarized.

Key words: Bayesian Estimation, Collateral Information, Differential Strategies, Empirical Bayes Estimation, Information Matrices, Item Response Theory, Missing Data

Introduction

Item response theory (IRT) models in psychometrics give the probability that an examinee will respond correctly to a given test item in terms of parameters for just that examinee and that item. This formulation makes it possible to solve many practical measurement problems that are difficult or intractable under classical test theory, including adaptive ability testing, large population equating studies, and test construction to targeted operating specifications.

It is standard practice to estimate IRT item parameters solely from the observed responses of a sample of examinees. This project was motivated by a desire to improve estimation by exploiting collateral information that is often available about test items (e.g., format, content, or cognitive processing requirements) or about examinees (e.g., educational background or demographic status). Table 1 lists the reports from the project exploring both practical and theoretical aspects of the problem. The present report summarizes the main results. The interested reader is referred to the individual papers for details, derivations, and examples.

Table 1 about here

Incorporating Collateral Information into IRT

The initial thrusts of the project were to determine how to incorporate collateral information into estimation procedures when the IRT model is correct, and to gauge its impact on estimation precision. Bayesian and empirical Bayesian methods were employed to this end. This section describes the basic model (Mislevy, 1987; in press).

Under an IRT model, the probability of response x_j to Item j with a possibly vector-valued item parameter β_j from an examinee with proficiency parameter θ is given as

$$P(x_j|\theta,\beta_j) = f(x_j|\theta,\beta_j) \quad , \tag{1}$$

where the form of the item response function f is known up to the item parameters. Under the usual assumption of local independence, the conditional probability of the response pattern $\underset{\sim}{x} = (x_1,\ldots,x_n)$ to n test items is simply the product of expressions like (1):

$$P(\underset{\sim}{x}|\theta,\underset{\sim}{\beta}) = \prod_j P(x_j|\theta,\beta_j) \quad , \tag{2}$$

where $\underset{\sim}{\beta} = (\beta_1,\ldots,\beta_n)$. Let the data matrix $\underset{\sim}{X} = (\underset{\sim}{x}_1,\ldots,\underset{\sim}{x}_N)$ represent response vectors observed from a sample of N examinees from a population in which θ follows the density $p(\theta)$. The likelihood for $\underset{\sim}{\beta}$ induced by $\underset{\sim}{X}$ is obtained as

$$L_x(\underset{\sim}{\beta}|\underset{\sim}{X}) = \prod_i \int f(\underset{\sim}{x}_i|\theta,\underset{\sim}{\beta})\ p(\theta)\ d\theta \ . \qquad (3)$$

"Marginal maximum likelihood" (MML) estimates of item parameters (e.g., Bock and Aitkin, 1981) are obtained by maximizing (3) with respect to $\underset{\sim}{\beta}$.

Suppose that in addition to item responses, values of collateral variables y are also available from examinees. The appropriate marginal likelihood is now

$$L_{xy}(\underset{\sim}{\beta}|\underset{\sim}{X},\underset{\sim}{Y}) = \prod_i \int f(\underset{\sim}{x}_i|\theta,\underset{\sim}{\beta})\ p(\theta|y_i)\ d\theta \ . \qquad (4)$$

MML estimates of item parameters that exploit collateral information about examinees are obtained by maximizing (4) with respect to $\underset{\sim}{\beta}$ (Mislevy, 1987).

Bayesian item parameter estimates are obtained from posterior distributions for $\underset{\sim}{\beta}$, which arise as the normalized product of a likelihood function such as (3) or (4) and a prior distribution for $\underset{\sim}{\beta}$, say $g(\underset{\sim}{\beta})$. If, before observing data, one possesses no information to differentiate expectations about the parameters of different items, an <u>exchangeable</u> prior for $\underset{\sim}{\beta}$ is appropriate; that is, the items are modeled as if they were n random draws from the same distribution. In this case the posterior distribution is given by

$$p_x(\underset{\sim}{\beta}|\underset{\sim}{X}) \propto L_x(\underset{\sim}{\beta}|\underset{\sim}{X}) \prod_j g(\beta_j) \tag{5}$$

or

$$p_{xy}(\underset{\sim}{\beta}|\underset{\sim}{X},\underset{\sim}{Y}) \propto L_{xy}(\underset{\sim}{\beta}|\underset{\sim}{X},\underset{\sim}{Y}) \prod_j g(\beta_j) \ , \tag{6}$$

depending on whether collateral information is available about examinees. If values on the collateral variable z are additionally available about items, they are incorporated as

$$p_{xz}(\underset{\sim}{\beta}|\underset{\sim}{X},\underset{\sim}{Z}) \propto L_x(\underset{\sim}{\beta}|\underset{\sim}{X}) \prod_j g(\beta_j|z_j) \tag{7}$$

or

$$p_{xyz}(\underset{\sim}{\beta}|\underset{\sim}{X},\underset{\sim}{Y},\underset{\sim}{Z}) \propto L_{xy}(\underset{\sim}{\beta}|\underset{\sim}{X},\underset{\sim}{Y}) \prod_j g(\beta_j|z_j) \tag{8}$$

(Mislevy, in press). Standard Bayesian procedures for estimating item and population parameters that do not employ collateral information extend to (7) and (8) in a straightforward manner (Mislevy, 1987, in press).

Increase in Information: Theoretical Results

Using general results about missing data problems, such as Orchard and Woodbury's (1972) "missing information principle," it is possible to derive upper and lower bounds for the expected precision of item parameter estimates with and without collateral

information (Mislevy and Sheehan, 1988, in press). The results are expressed most easily in Bayesian terms.

Consider first the impact of collateral information about <u>examinees</u>. Let $V(\boldsymbol{\beta}|\boldsymbol{\theta},\boldsymbol{X},\boldsymbol{Y})$ represent the posterior variance of $\boldsymbol{\beta}$ that would be obtained after observing values of not only item responses x and collateral variables y from a sample of N examinees, but values of their latent proficiencies θ as well. Let analogous expressions represent posterior variance of $\boldsymbol{\beta}$ when values of one or more types of variables are not observed; for example, $V(\boldsymbol{\beta}|\boldsymbol{X})$ when only item responses are observed. The following relationships may be derived:

$$
\begin{aligned}
E[V(\boldsymbol{\beta}|\boldsymbol{\theta},\boldsymbol{X},\boldsymbol{Y})] &= E[V(\boldsymbol{\beta}|\boldsymbol{\theta},\boldsymbol{X})] \\
&\leq E[V(\boldsymbol{\beta}|\boldsymbol{X},\boldsymbol{Y})] \\
&\leq E[V(\boldsymbol{\beta}|\boldsymbol{X})] \ ,
\end{aligned}
$$

where $\boldsymbol{A}\leq\boldsymbol{B}$ means that the matrix difference $\boldsymbol{B}-\boldsymbol{A}$ is at least positive semidefinite. Thus the precision of item parameter estimation when using collateral information about examinees along with item responses is at least as great as that expected when using item responses alone, but cannot exceed the precision that would be expected with the same sample size if values of the latent variable θ could be observed as well.

An obvious lower bound holds the impact of collateral information about <u>items</u>:

$$E[V(\underset{\sim}{\beta}|\underset{\sim}{X},\underset{\sim}{Z})] \leq E[V(\underset{\sim}{\beta}|\underset{\sim}{X})] \;;$$

that is, expected precision when using collateral information about items in addition to item responses, equals or exceeds precision expected when not using it. No ordering holds between $E[V(\underset{\sim}{\beta}|\underset{\sim}{X},\underset{\sim}{Z})]$ and $E[V(\underset{\sim}{\beta}|\underset{\sim}{\theta},\underset{\sim}{X})]$ in general. In particular, when $\underset{\sim}{Z}$ is employed along with $\underset{\sim}{X}$, it is possible to exceed the precision obtainable with $\underset{\sim}{\theta}$ and $\underset{\sim}{X}$.

Increase in Information: Practical Results

By examining the structure of information matrices with and without collateral information, and by applying the methods to data from the National Assessment of Educational Progress (NAEP) and the Profile of American Youth surveys, it was found that modest increases in the precision of item parameter estimates can be achieved by using collateral information (Mislevy, 1987, in press; Mislevy and Sheehan, 1988, in press).

From collateral information about <u>examinees</u>, increases in information depend on the strength of the relationship of the collateral variables with θ. In typical educational and psychological settings where collateral information can often account for about a third of the population variance, and with item reliabilities typical of those settings, gains equivalent to 2 to 6 additional test items can be expected. This gain is substantial when few responses are available from each examinee, as in educational assessments, and may be useful in adaptive testing where tests are short but well-targeted. It is

unimpressive in individual achievement testing, where tests of sixty items or more are common.

From collateral information about _items_, increases equivalent to hundred and fifty additional examinees were found for Rasch item difficulty parameters in a junior high fractions test (Mislevy, in press). While a gain of this magnitude would be unimpressive in applications where data from thousands of examinees is already at hand, it is meaningful in situations when either (1) few examinees have been tested, as in the fractions example or in local testing problems, or (2) _no_ examinees have been tested, as when approximating item statistics for newly-written test items.

In addition to small-sample applications, collateral information about _items_ can play an important role in both item construction and diagnosis regardless of sample size. The conditional distributions of item parameters, $p(\beta|z)$, express item operating characteristics such as difficulty in terms of salient features of the items. To the degree that these distributions succeed in explaining item operating characteristics, the test constructor can manipulate the features to modify items in intended ways or to create new items that tap the same essential skills. To the degree that items depart from the centers of these predictive distributions, they are hard or easy for reasons other than those held most important in describing the domain. Outliers are suspect as flawed or irrelevant. _The approach implied by (5) and (6) is a step in the direction of integrating educational and_

psychological theory into the measurement process. (Its application to the items in the Document Utilization scale of the NAEP Survey of Adult Literacy is currently in progress.)

When Collateral Information Must Be Used

The preceding sections discuss how, when all examinees are presented all items, collateral information about examinees and items may be exploited to obtain more precise item parameter estimates. Consistent estimates are still obtained in this case if the collateral information is not used (Mislevy and Sheehan, in press). The same results apply when each examinee receives only a random subset of items.

This is not the case that obtains in many practical applications of IRT, however. In order to obtain more information about item or examinee parameters per observed response, items are often administered to examinees as a function of item and examinee collateral variables. Fourth grade students may be presented an easier test form than the overlapping form fifth graders receive, for example; and a high school graduate may be presented a harder item first in an adaptive test than a nongraduate. In order to obtain consistent MML item parameter estimates, it is mandatory to employ collateral information about examinees--i.e., to use (4) rather than (3) (Mislevy and Sheehan, in press). In order to obtain the correct Bayesian inferences, it is mandatory to use collateral information about items as well--i.e., to base inferences on (8) rather than (4) (Mislevy and Wu, 1988). Mislevy

and Sheehan (in press) give a simple counterexample with the Rasch model to demonstrate an asymptotic bias in item parameter estimation in such a case if collateral information is ignored.

Modeling Item Responses when Different Examinees Follow Different Solution Strategies

Initial work on using collateral information about items assumed that the IRT model was strictly correct. Thinking about the features of items that made them easy or hard, however, made it clear that difficulty depends on the way that the examinees are attempting to arrive at their answers. In particular, different features of items can make them differentially difficult for examinees who follow different solution strategies. This insight led to the formulation of a mixture of IRT models (Mislevy and Verhelst, in press). Resolving the mixture demands a type of collateral information that plays no role whatsoever in traditional psychometrics, including standard IRT: psychological theory about the different strategies that examinees might follow.

The key idea is to model item difficulty in terms of salient item features--features that tend to make an item easy or difficult under various strategies. The Mislevy-Verhelst model makes the following assumptions:

1. A finite number of known solution strategies apply.

2. Each examinee is applying the only one of these strategies for all the items in the set.

3. The responses of an examinee are observed but the strategy he or she has employed is not.

4. The responses of examinees following Strategy k conform to an item response model of a known form.

5. Substantive theory posits relationships between observable features of items and the probabilities of success enjoyed by members of each strategy class. The relationships may be known either fully or only partially--e.g., known as to parametric form but not parameter values.

Let $\theta = (\theta_1, \ldots, \theta_K)$ be an examinee proficiency parameter, with the element θ_k corresponding to proficiency if Strategy k is employed. Let $\phi = (\phi_1, \ldots, \phi_K)$ be an examinee strategy parameter, with all elements zero except for the single element k corresponding to the strategy that is employed; this element takes the value 1. Let the operating characteristics of Item j under Strategy k be given as follows:

$$P[x_j | \theta_k, \beta_k(z_{jk}|\alpha), \phi_k = 1] = f_k[x_j | \theta_k, \beta_k(z_{jk}|\alpha)] \ , \qquad (9)$$

where $\beta_k(z_{jk}|\alpha)$, the item parameter for Item j that applies when examinees follow Strategy k, depends on its salient features z_{jk} under that strategy and a relatively small number of basic strategy parameters α. The MML function for estimating α induced by the data matrix $\underset{\sim}{X}$ from a sample of N examinees and the item/strategy collateral variables $\underset{\sim}{Z}$ is obtained as

$$L(\alpha|\underset{\sim}{X},\underset{\sim}{Z}) = \prod_{i=1}^{N} \sum_{k=1}^{K} \pi_k \prod_{j=1}^{n} f_k[x_{ij}|\theta,\beta_k(z_{jk}|\alpha)]\ g_k(\theta)\ d\theta\ , \quad (10)$$

where g_k is the density of θ_k among those examinees following Strategy k, and π_k is the proportion of the population who do so. If the g_ks and the πs are not known, they too can be estimated via MML by maximizing (10) with respect to them as well.

If the αs, g_ks, and πs are known or well estimated, it is possible to calculate for a given examinee the probability that his response vector was produced under a given strategy and to estimate his ability under each possibility. By Bayes theorem, the posterior probability of Strategy k and proficiency θ under that strategy is obtained as

$$P(\theta,\phi_k=1|\underset{\sim}{x}) = C \quad f_k(\underset{\sim}{x}|\theta,\beta_k(z_{jk}))\ g_k(\theta)\ \pi_k \quad ,$$

where C is the normalizing constant obtained as

$$C^{-1} = \sum_{k} \int f_k(\underset{\sim}{x}|\theta,\beta_k(z_{jk}))\ g_k(\theta)\ d\theta\ \pi_k \quad .$$

The posterior probability that Strategy k was employed is

$$P(\phi_k=1|\underset{\sim}{x}) = \int P(\theta,\phi_k=1|\underset{\sim}{x})\ d\theta$$

and the posterior mean proficiency conditional on $\phi_k=1$ (i.e., supposing that Strategy k was used) is

$$E(\theta_k | \underset{\sim}{x}, \phi_k = 1) = \int \theta \, P(\theta, \phi_k = 1 | \underset{\sim}{x}) \, d\theta \quad P^{-1}(\phi_k = 1 | \underset{\sim}{x}) \; .$$

<u>The significance of this model lies in its ability to express how examinees solve items rather than just how many they solve</u>. The latter is all that the standard models of test theory can do. Areas of potential benefit include psychological investigations of alternative processing models, educational decisions involving level of understanding, and determinations of alternative mental models in problem solving. The approach opens the door to such applications as (1) adaptive testing schemes designed to infer how examinees solve problems as well as how well they solve them, and (2) studies of changes in the structure as well as the level of intelligence in the course of human development.

Inferring Examinee Ability When Some Item Responses Are Missing

In practical applications of item response theory (IRT), there are several reasons that item responses may not be observed from all examinees to all test items. The reason most germane to the collateral information problem is the intentional administration of only subsets of items to examinees, with the subset depending on collateral information. It was mentioned above that collateral information <u>must</u> be taken into account in these cases. In addition to this type of missingness, Mislevy and Wu (1988) studied problems of inference that arise with several other types of missingness that arise frequently in IRT.

To preface the results of their study, we review Rubin's (1976) notions about "ignorability" of missing data. Ignoring the

missingness process under direct likelihood inference means using a pseudo-likelihood that includes terms for only the responses that were observed, without regard for the processes by which they came to be observed. The resulting inferences are appropriate if the pseudo-likelihood is proportional to the correct likelihood that does account for the missingness process. In this case the correct point estimate of the maximum likelihood estimate (MLE) is obtained. Sampling-distribution inferences based on the MLE are appropriate only if the missingness pattern does not depend on the values of the observed data. When this condition holds, sampling-distribution inferences can be drawn with regard to repeated samples of responses to only those items whose responses were observed. The missingness process is ignorable with respect to Bayesian inference if the correct Bayesian posterior is proportional to the product of the pseudo-likelihood and an appropriate prior distribution.

For fives common types of missingness in IRT, Mislevy and Wu first used Rubin's (1976) theorems to determine whether ignorability holds under direct likelihood and Bayesian inference about examinee parameters θ when item parameters $\underset{\sim}{\beta}$ are known. In those cases in which the correct value of the MLE is obtained under direct likelihood inference, they asked whether sampling distribution inferences based on the MLE were appropriate. They then considered the analogous questions for inferences about $\underset{\sim}{\beta}$ when the examinee parameters are eliminated by marginalization, as

in (3)-(8). The findings are summarized below. Tables 2 and 3 highlight the results on ignorability.

Tables 2 and 3 about here

Case 1: Alternate Test Forms. When an examinee is assigned one of several alternative test forms by a random process such as a coin flip or a spiralling scheme, the process that renders missing the responses to items on the forms not presented is ignorable for all three types of inference, both for estimating $\underset{\sim}{\beta}$ and for estimating θ when $\underset{\sim}{\beta}$ is known.

Case 2: Targeted Testing. When collateral variables such as educational or demographic status are used to assign an examinee one of several test forms that differ in their measurement properties, the resulting missingness on forms not given is ignorable under direct likelihood inference for θ given $\underset{\sim}{\beta}$, but not under Bayesian inference unless the prior information about examinees that led to differential assignments is conditioned on. This information must be taken into account for both likelihood and Bayesian inferences about $\underset{\sim}{\beta}$; for Bayesian inference, prior information about $\underset{\sim}{\beta}$ used to select items must additionally be taken into account. Sampling distribution inferences may be based on MLEs for $\underset{\sim}{\beta}$ and for θ given $\underset{\sim}{\beta}$, conditional on the observed patterns of form administration within values of the examinee variables used for targeting.

It should be emphasized that these conclusions depend on the veracity of the IRT model. In particular, it is necessary that the regression of a correct response on ability be invariant with respect to collateral information. This assumption may well fail in a situation of currently increasing interest: An item pool is calibrated using an IRT model, and a school is allowed to measure students using only those items it deems relevant to its curriculum. If students from different schools have had different opportunities to learn the skills tapped by different items, then tailoring tests to their strengths leads almost certainly to item by school by ability interactions--a violation of the IRT model. Estimates for schools and individuals within schools tend to overestimate the scores they would have received had they been given all items, or randomly selected subsets of items. This use of IRT may hold practical value nonetheless, provided that such scores are viewed not as consistent estimates of performance in the total pool but as indicators of a kind of maximal performance.

Case 3: Adaptive Testing. In adaptive testing, item assignment proceeds item by item for each examinee according to the values of his responses to preceding items. The same conclusions as for Case 2 hold for direct likelihood and Bayesian inference. Ignorability under direct likelihood inference means that the correct points are identified as MLEs of θ given $\underset{\sim}{\beta}$ and of $\underset{\sim}{\beta}$. The usual MLE properties under sampling-distribution inference need not hold, however, because the probabilities of missingness patterns depend on the values of observed responses.

Case 4: Not-reached Items. When some examinees run out of time before they see the last items on a nearly nonspeeded test, the not-reached process is ignorable with respect to direct likelihood inference about θ given $\underset{\sim}{\beta}$, and the MLE supports sampling distribution inferences that pertain to repeated administrations of the items that were actually reached. This missingness process is not ignorable under Bayesian inference unless speed and ability are independent. And only then can direct likelihood inferences about $\underset{\sim}{\beta}$ ignore the missingness. Furthermore, Bayesian inferences about $\underset{\sim}{\beta}$ require that collateral variables for items be employed if they played a role in determining which items would not be reached, as when items are ordered from easy to hard.

Case 5: Intentional Omission. When examinees are presented items, have a chance to appraise their content, and decide for their own reasons not to respond, the missingness is not ignorable. Inferences must be drawn from a full model for the joint distribution of missingness and item response.

Not surprisingly, modeling this nonignorable nonresponse is difficult. Neither of the two most ambitious approaches proposed to date, namely Lord's (1983) model for omits and the use of multiple-category IRT models (e.g., Bock, 1972), handles the issue of local independence in a fully satisfactory manner. Under Lord's (1983) model, the marginal model for item responses is not a standard IRT model depending on θ alone and exhibiting local independence. Under the multiple-category model approach, local

independence fails unless all examinees at any given ability level have the same propensity to omit items they are unsure of, rather than guess at random.

If one assumes that examinees are perfect judges of their chances of responding correctly, and omit only if it is in accordance with the strategy that maximizes their expected score, Lord's (1974) treatment of omits as fractionally correct can be justified as providing the expectation of a conditional term in the full likelihood for omission probabilities and correct-response probabilities. This procedure is readily incorporated into standard complete-data IRT algorithms and avoids having to specify the full likelihood, but sacrifices information about examinee and item parameters conveyed by the observed pattern of missingness. Given the complexity of models for the full likelihood, however, this expedient seems to be a good practical choice--provided that, as Lord urges, examinees are clearly informed about how omits will be scored and which omitting strategy maximizes their chances of scoring well.

Conclusion

Although collateral information about examinees and items is rarely employed in item response theory (IRT), it is straightforward to incorporate it using Bayesian and empirical Bayesian methods. If the IRT model is correct and examinees are assigned items independently of values on collateral variables, then collateral information can be used to improve item parameter estimation modestly. Employing collateral information is

mandatory to obtain correct Bayesian and empirical Bayesian inferences if it was used to assign items to examinees.

Aside from considerations of efficiency, employing collateral information about items is a step toward integrating educational and psychological theory into the measurement process. Two aspects of this idea were developed in the course of the project.

The first, which takes a more traditional measurement perspective, assumes that a single IRT model provides an acceptable fit to the data of interest. Modeling items' operating characteristics in terms of salient features can make estimation more precise, but more importantly it elucidates the reasons that items are hard or easy, and why some are more discriminating than others. A formal framework is thus available for item construction and diagnosis, expressing relationships among substantive theory, item features, and measurement properties.

The second is a response to a growing awareness of the fact that traditional psychometric models (IRT as well as classical test theory) measure what is essentially an overall level of proficiency--losing in the process qualitative differences among examinees that arise from different cognitive solution strategies. In order to extend psychometric analysis to these problems, and to bring to bear the findings of recent research upon applied measurement problems, it is _mandatory_ to employ collateral information about examinees and items that bears upon the ways that people solve problems. A mixture of IRT models that applies to some problems of this type was introduced in the project.

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Table 1

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Table 2

Ignorability Results for Estimating θ Given $\underset{\sim}{\beta}$

Type of Missingness	Type of Inference: Direct Likelihood	Bayesian	Sampling Distribution*
Alternate Forms	Yes	Yes	Yes
Targeted Forms	Yes	Yes, given examinee variables	Yes
Adaptive Testing	Yes	Yes, given examinee variables if they are used	No
Not-Reached	Yes	No, unless speed and ability are independent	Yes
Intentional Omissions	No	No	No

* Conditional on the observed pattern of missingness.

Table 3

Ignorability Results for Estimating $\underset{\sim}{\beta}$ After Marginalizing over θ

Type of Missingness	Type of Inference: Direct Likelihood	Bayesian	Sampling Distribution*
Alternate Forms	Yes	Yes	Yes
Targeted Forms	Yes, given examinee variables	Yes, given examinee and item variables	Yes, given examinee variables
Adaptive Testing	Yes, given examinee variables if they are used	Yes, given item variables and examinee variables if they are used	No
Not-Reached	No, unless speed and ability are independent	No, unless speed and ability are independent	No, unless speed and ability are independent
Intentional Omissions	No	No	No

* Conditional on the observed pattern of missingness.

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